practitioners of quality control who are interested in a comprehensive account of sampling inspection as well as in the procedures and tables for its application.

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50[K].—C. W. DUNNETT & R. A. LAMM, "Some tables of the multivariate normal probability integral with correlation coefficients $\frac{1}{3}$," Lederle Laboratories, Pearl River, New York. Deposited in UMT File.

The probability integral of the multivariate normal distribution in n dimensions, having all correlation coefficients equal to ρ (where necessarily $-\frac{1}{n-1} < \rho < 1$), is given by

$$\int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_m} \left(\frac{1}{2\pi}\right)^{n/2} \frac{[1 + (n-1)\rho]^{-1/2}}{(1-\rho)\,\frac{(n-1)}{2}} \exp\left[\frac{1 + (n-2)\rho}{(1-\rho)[1+(n-1)\rho]} \cdot \left\{\sum x_i^2 - \frac{2\rho}{1+(n-2)\rho} \sum_{i \neq j}\sum x_i x_j\right\}\right] dx_1 \cdots dx_n$$

This function, which we shall denote by $F_{n,\rho}(x_1, \cdots, k_n)$, has been tabulated for $\rho = \frac{1}{2}$ and $x_1 = \cdots = x_n$ by Teichroew [1]. In the present paper, we present a table for the case $\rho = \frac{1}{3}$ and $x_1 = \cdots = x_n$. The need for this table arose in connection with a multiple-decision problem considered by one of the authors [2].

In computing the table, use was made of the fact that, for $\rho \geqq 0$, $F_{n,\rho}(x_1, \cdots, x_n)$ belongs to a class of multivariate normal probability integrals which can be written as single integrals (see Dunnett and Sobel [3]), a fact which greatly facilitates their numerical computation. In this case, we have

$$F_n, \rho(x_1, \cdots, x_n) \equiv \int_{-\infty}^{+\infty} \prod_{i=1}^{n} \left[F\left(\frac{x_i + \sqrt{\rho}y}{\sqrt{1-\rho}}\right)\right] f(y)\, dy$$

where

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \quad \text{and} \quad F(y) = \int_{-\infty}^{y} f(y)\, dy.$$

The attached table was computed by replacing the right-hand side of this identity by the series based on the roots of Hermite polynomials described by Salzer *et al.* [4]. Those tabular values marked with an asterisk have been checked by comparison with the values obtained by applying Simpson's rule. The values checked were found to be systematically less than the Simpson's rule values by an amount which varied between .0000000 and .0000013, depending on n. This indicates that the error in the tabular values may be no more than 1 or 2 units in 6th decimal place, but further checks are required in order to substantiate this.

The table gives $F_{n,1/3}(x, \cdots, x)$ to six decimal places, with x varying from 0 to $7.0/\sqrt{3}$ in steps of $0.1/\sqrt{3}$ for $n = 1\ (1)\ 10$, and from $1.5/\sqrt{3}$ to $2.1/\sqrt{3}$ in steps of $0.01/\sqrt{3}$ for $n = 1\ (1)\ 10, 13, 18$.

AUTHORS' ABSTRACT

1. D. TEICHROEW, "Probabilities associated with order statistics in samples from two normal populations with equal variance," Chemical Corps Engineering Agency, Army Chemical Center, Maryland, 1955.
2. C. W. DUNNETT, "On selecting the largest of k normal population means," (to be published in *Jn.*, Roy. Stat. Soc, Series B, 1960).
3. C. W. DUNNETT & M. SOBEL, "Approximations to the probability integral and certain percentage points of a multivariate analogue of Student's t-distribution," *Biometrika*, v. 42, 1955, p. 258.
4. H. E. SALZER, R. ZUCKER & R. CAPUANO, "Table of the zeros and weight factors of the first twenty Hermite polynomials," *Jn. Res.*, Nat. Bur. Standards, v. 48, 1952, p. 111.

51[K].—E. C. FIELLER, H. O. HARTLEY & E. S. PEARSON, "Tests for rank correlation coefficients. I," *Biometrika*, v. 44, 1957, p. 470–481.

This paper is concerned with sampling determination of the approximate distribution for $z_S = \tanh^{-1} r_S$ and $z_K = \tanh^{-1} r_K$, where r_S is Spearman's rank correlation coefficient and r_K is Kendall's rank correlation coefficient, for the case of sample of size n from a bivariate normal distribution. It is concluded that z_S and z_K are approximately normally distributed if n is not too small, with var $(z_S) \doteq 1.060/(n - 3)$ and var $(r_K) \doteq 0.437/(n - 4)$. Eight tables are presented. Table 1 contains 4D values of three versions of var (r_S) for $\rho = 0.1(0.1)0.9$ and $n = 10, 30, 50$; one version is Kendall's approximate formula (adjusted), another is the observed value, and the third is a smoothed form of the observed value. Table 2 contains 3D values of var $(r_S)/[1 - (Er_S)^2]$ and 4D values of var $(r_K)/[1 - (Er_K)^2]$, also an average over ρ for each of these, for $\rho = 0.1(0.1)0.9$ and $n = 10, 30, 50$. Table 3 contains 3D approximate theoretical and observed values for Ez_S, while Table 4 contains these values for Ez_K, where $\rho = 0.1(0.1)0.9$ and $n = 10, 30, 50$; the second-order correction terms for the theoretical values are also stated to 3D. Table 5 contains 4D values of the observed variance of z_S and 3D values of its observed standard deviation, likewise for Table 6 with z_K, where $\rho = 0.1(0.1)0.9$ and $n = 10, 30, 50$. Table 7 contains values of χ^2 for goodness of fit tests of the normality of z_S and z_K for $n = 30, 50$. Table 8 contains 2D and 3D values of

$$(Ez_1 - Ez_2)/\sqrt{\text{var } (z_1) + \text{var } (z_2)}$$

for z_1 and z_2 representing the same correlation coefficient but with different ρ values ($\rho_2 = \rho_1 + 0.1$); this is for the product moment correlation coefficient, Spearman's coefficient and Kendall's coefficient with $\rho_1 = 0.1(0.1)0.8$ and $n = 10, 30, 50$.

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52[K].—G. HORSNELL, "Economical acceptance sampling schemes," Roy. Stat. Soc., *Jn.*, sec. A., v. 120, 1957, p. 148–201.

This paper is concerned with acceptance sampling plans designed to minimize the effective cost of accepted items produced under conditions of normal production. Effective cost per accepted item is defined to be the production cost per lot plus the average cost of inspection per lot when apportioned equally over the average number of items accepted per lot from production of normal quality. Single-sample plans are examined in detail. Double-sample plans are considered briefly.

An appendix contains thirty-one separate tables for single-sampling plans,